In metrology, discrepancies occasionally occur when different techniques are used to measure the same property. When these differences become large, critical examination and validation of the methods become necessary. Two properties in the paper industry that are subject to large measurement discrepancies are caliper and bending stiffness, both of which are measured by several different instruments. The methods of measuring thickness reviewed by Fellers et al. [1] include hard-platen, effective-thickness, mercury-displacement, spherical, and soft-platen methods.

Bending stiffness can be calculated [2] from measurements of soft caliper and tensile stiffness. Comparison of this theoretical bending stiffness with experimental bending-stiffness measurements can highlight the validity or shortcomings of a given experimental method. A variety of instruments exist for measuring bending stiffness [3–5]. The two most common instruments in use are the Lorentzen & Wettre (L&W) bending-resistance tester and the Taber stiffness tester. A study by Fellers [6], modeled after an earlier study by Verseput [7], examined the coefficient of variation of bending stiffness with in and between laboratories for a range of paper and paperboard grades. Their results showed that the L&W stiffness tester had a lower coefficient of variation than the Taber stiffness tester. Another study by Koran and Kamden [3] showed that the bending stiffness calculated from tensile stiffness has a linear relationship to the Taber stiffness, with $r^2 = 0.90$.

This paper re-examines the differences between the Taber and L&W stiffness measurements and compares the results with theoretical bending stiffness values calculated from tensile modulus measurements. This paper also looks at the effect of estimating the thickness of a single sheet from a stack of sheets comparing the resulting thickness value to those obtained from either soft or hard platen measurements and to an effective thickness calculation as proposed by Setterholm [8]. The exercise can either instill confidence in the measurement techniques or reveal limitations in the measurement methods. Stiffnesses and moduli obtained mechanically are also compared with values measured by ultrasonic techniques to confirm the relative ranking of sample values.

### Table I: Samples used in this study: basis weights, soft-platen calipers, and densities.

<table>
<thead>
<tr>
<th>Sample ID</th>
<th>Description</th>
<th>Basis Weight (g/m²)</th>
<th>Soft Caliper (μm)</th>
<th>Density (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>42# Brown kraft linerboard</td>
<td>212</td>
<td>277</td>
<td>766</td>
</tr>
<tr>
<td>B</td>
<td>26# Neutral sulfite semi-chemical medium</td>
<td>130</td>
<td>191</td>
<td>679</td>
</tr>
<tr>
<td>C</td>
<td>Lightweight bleached Kraft</td>
<td>75</td>
<td>92</td>
<td>823</td>
</tr>
<tr>
<td>D</td>
<td>Newsprint</td>
<td>45</td>
<td>57</td>
<td>808</td>
</tr>
<tr>
<td>E</td>
<td>Lightweight coated paper</td>
<td>47</td>
<td>44</td>
<td>1057</td>
</tr>
<tr>
<td>F</td>
<td>Mylar transparency film</td>
<td>146</td>
<td>98</td>
<td>1500</td>
</tr>
<tr>
<td>G</td>
<td>Synthetic paper</td>
<td>156</td>
<td>97</td>
<td>1610</td>
</tr>
</tbody>
</table>

The exercise can either instill confidence in the measurement techniques or reveal limitations in the measurement methods. Stiffnesses and moduli obtained mechanically are also compared with values measured by ultrasonic techniques to confirm the relative ranking of sample values.

### EXPERIMENTAL

Seven different samples representing a range of properties were selected for investigation. Table I shows the designa-
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...tion, grade, basis weight, soft caliper, and density for each sample used in this study.

All sheets were environmentally conditioned before measurement according to TAPPI method T402 sp-03. These samples were used to generate values for hard caliper, soft caliper, a “stack” caliper, and the MD and CD calculated effective thicknesses. The hard and soft calipers were based on TAPPI methods T411 om-05 and T551 om-06, respectively. The hard caliper was measured on an Emveco 200A, and the soft caliper was measured on an Emveco 210-DH. The 210-DH was calibrated for each sample with an appropriate calibration shim before testing. The “stack” caliper was measured by stacking twelve specimens of a sample and dividing the result by twelve.

The effective mechanical equivalent thickness ($t_e$) was also calculated for comparison of results. This parameter, proposed by Setterholm, calculates the thickness from the relationship between tensile stiffness ($S_t$) and bending stiffness ($S_b$) [8], as follows:

$$S_t = E_{ij} \cdot t$$

$$S_b = E_{ij} \cdot \frac{t^3}{12}$$

$$t_e = \sqrt{\frac{12S_b}{S_t}}$$

where $E_{ij}$, or more generally $E$, is the in-plane elastic modulus, MD or CD, $E_{MD}$ or $E_{CD}$, or equivalently $E_{11}$ or $E_{22}$ in another standard notation. In addition to the caliper, the apparent density can also be calculated as:

$$\rho = \frac{G}{I} \cdot \left( \frac{\text{kg}}{\text{m}^4} \right)$$

where $\rho$ is the apparent density, $t$ is the soft caliper, and $G$ is the basis weight. Generally, the soft caliper is equal to the mechanical equivalent caliper, and this will be verified in this paper. Units of the parameters or the left-hand side of equations are given in parentheses.

The basis weight of each sample was averaged over 12 test specimens. We measured mass with an Ohaus Analytical Plus balance accurate to 0.01 mg. We used TAPPI method T410 om-98 to estimate the basis weight. Samples for basis weight were cut using an L&W pneumatic-punch sample cutter, model FI 107, and had an area of 33 cm$^2$. Basis weight was calculated according to the formula:

$$G = \frac{M}{A} \cdot \left( \frac{\text{kg}}{\text{m}^2} \right)$$

where $G$ is the grammage, $A$ is the specimen test area, and $M$ is the mass of the test specimen.

For bending-stiffness tests, samples were cut to 38 mm by 50 mm, using a Taber cutter, model 104-11, according to TAPPI method T566 om-97. The instruments used were the Lorentzen & Wettre 10-1 and the Taber 150-E. Averages for these tests in the MD and CD were calculated for five test specimens. The bending stiffness ($S_b$) had to be calculated from the Taber instrument digital readout and is actually the bending moment $M$. For two-point bending of a beam, standard Euler beam-mechanics analysis provides the relationship:

$$EI = \frac{M \theta}{3 \alpha}$$

where $I$ is the moment of inertia, $M$ the bending moment, and $\theta$ the length of the beam. Thus, the bending stiffness with standard units mN-m can be converted from the Taber moment readout in gf-cm to bending stiffness using:

$$S_b (\text{mN} \cdot \text{m}) = M \left( \frac{\text{gf} \cdot \text{cm}}{\text{mm}} \right) \cdot 9.8 \times 10^{-2} \left( \frac{\text{mN} \cdot \text{m}}{\text{gf} \cdot \text{cm}} \right) \cdot \frac{1}{l (\text{mm})} \cdot 60^\circ$$

where $l$ is the bending-stiffness moment test span (50 mm), $w$ is the specimen width (38.1 mm), and $\alpha^\circ$ is the bending angle in degrees ($15^\circ$). On the L&W, the bending stiffness was calculated using the two-point bending formula:

$$S_e (\text{mN} \cdot \text{m}) = \frac{60^\circ \cdot F (\text{N}) \cdot l^2 (\text{mm}^2)}{\pi \cdot w (\text{mm}) \cdot \alpha^\circ}$$

where $F$ is the force measured at the moment point by the L&W load cell and read out on the numeric display. The instrument has a direct readout calibrated in mN-m only for the case of a 50-mm testing span and a five-degree bending angle. Otherwise, for other testing parameters, the readout is $F$ in mN, and Eq. (6) is used. A caveat is that this formula assumes that beam strains are in the linear elastic region of the stress-strain curve for the material. This requires that the specimen deflection be small enough that the applied moment point curves around the beam in a circular arc, and that the test specimen be idealized as a beam in the analysis reported in (6) is long compared with the thickness, so no transverse shear occurs. These assumptions are not necessarily valid for a given standard bending-stiffness measurement, and therefore the methods are subject to scrutiny. Examination of tensile-test stress-strain curves for the sample set showed that strains of less than 0.2% were within the linear elastic region of the curve. For the two-point bending stiffness measurement method, we can estimate the bending strain ($\varepsilon_e$) as:

$$\varepsilon_e = \frac{\pi \cdot l}{120 \cdot l} \cdot \alpha^\circ,$$
Table II shows a comparison of the angles, spans, and equivalent strains used in the L&W measurements for the seven samples. From examination of the stress-strain curves, it is clear that the stiffness values at 5° should be higher than those at 30°, because the elastic modulus for nonlinear strains is lower than that for linear strain, and at 30°, the calculated strains exceed the 0.2% limit for apparent linearity. Light-weight grades cannot be easily measured with the L&W instrument at the standard default deflection of 5º and standard load-cell sensitivity, so large angles are required, at the risk of underestimating the bending stiffness from nonlinear strain.

The tensile measurements were made on an Instron 1122 uniaxial testing frame according to TAPPI method T494 om-96. The elastic modulus was calculated by (8):

$$E = \frac{dF}{dx_{max}} \cdot \frac{L}{w \cdot t_s} \cdot (8)$$

The first term is obtained by using the Instron Series IX™ software tensile-stiffness slope-fitting algorithm and calculating the linear slope of the load $F$ versus displacement $x$ data which have been confirmed to be accurate through comparison with manual analysis of the raw data. $L$ is the gauge length or length of the unclamped portion of the test specimen (178 mm), $w$ is the width of the sample (25.4 mm), and $t$ is the soft caliper measured and entered separately.

Bending stiffness was calculated from the tensile-test load-displacement curve according to Eq. (9), with brackets indicating average quantities:

$$\left< S_{b} \right> = \frac{(E)t_s}{12} \left[ 1 \pm \left( \frac{\sigma_E}{(E)} \right)^2 + 3 \left( \frac{\sigma_t}{(t_s)} \right)^2 \right] \cdot (9)$$

The $\sigma_t$ are standard deviations, and the summation in the quadrature rule in (9) is used for the propagation of relative errors. The factor of three in the calculation of the relative error arises from the cube term in the bending-stiffness formula. Averages were calculated based on at least five repeat measurements. The bending stiffness used to compute the effective thickness in (1) was taken from the L&W 5° results.

### Ultrasonic measurements

In ultrasonic testing of paper, the orthotropic stiffness matrix can be reduced if the surfaces of the sheet are unrestrained. In that case, the surface stresses must be zero, and so the constitutive relation becomes:

$$\sigma_{11} = C_{11} - C_{12} - C_{0} = 0 \quad \sigma_{22} = C_{12} - C_{22} - C_{0} = 0 \quad \sigma_{12} = 0 \quad 2C_{0} = 2C_{0} \cdot \cdot \cdot (10)$$

The aim here is to obtain the elastic moduli by ultrasonic means. This can be done by relating the stiffness of a visco-elastic sheet to the square of the velocity using the following set of equations for the stiffnesses $C_{ij}$ [11]:

$$C_{11} = \rho \cdot c_{ls}^2 \quad C_{22} = \rho \cdot c_{ls}^2 \quad C_{46} = \rho \cdot c_{xy}^2$$

$$C_{12} = \left\{ 2 \rho \cdot c_{s45}^2 - \frac{1}{4}(C_{11} + C_{22}) - C_{46} \right\} \frac{1}{4} \left( C_{11} - C_{22} \right)^2 \Rightarrow C_{46}$$

where $\rho$ is the density of the sheet and $c$ is the velocity of the wave. The subscript $L$ denotes a longitudinal wave polarized and propagated down the MD, $L_y$ denotes the same in the CD, $S_{xy}$ denotes a shear wave polarized in the MD, but propagated in the CD (or vice versa), and $S_{45}$ denotes a shear wave polarized at 45° with respect to the MD, but propagated at -45° with respect to the MD.

The Poisson ratios $\nu_{ij}$ can be determined [12,13] from these four equations, and the elastic constants, MD and CD moduli $E_{MD}$ and $E_{CD}$, and shear modulus $G_{12}$ can be calculated as:
Ultrasonic wave-propagation measurements were made using the IPST-developed In-Plane Ultrasonic Robot. This instrument consists of a computer-controlled robot arm with its end effector fitted with a pair of sheet-contacting bi-planar “paddle” bimorph ultrasonic transducers whose spacing and orientation are pneumatically actuated for various types of propagating-pulse time-of-flight measurements. The direction of propagation of 80-kHz pulses for a measurement is determined by the effector’s rotation orientation with respect to the sheet MD. Shear waves, when required for Poisson ratio or shear modulus calculations, are generated along the sheet surface using an actuated 90° axial rotation of the paddle transducers. The time of flight of pulses along the sheet is recorded digitally and deconvolved to calculate the quantities of interest using the computer program. User-entered data required for the computer calculations for in-plane moduli and Poisson ratios are the soft caliper and basis weight of the test specimen. Ultrasonic moduli are a convenient and nondestructive means for comparing results with mechanical measurements.

RESULTS

Basis weight
The coefficient of variation, $C_V$, over several samples is shown in Fig. 1. This was calculated as:

$$C_V = \frac{\sigma}{\langle BW \rangle},$$  \hspace{1cm} (13)

where $\sigma$ is the standard deviation and $\langle BW \rangle$ is the arithmetic mean of the basis weights. Note that the $C_V$ stabilizes after five measurements, suggesting that the standard TAPPI method requirement for ten repeats may be unnecessary in many cases.

Caliper and density
Figure 2 shows the caliper of the different grades measured five different ways: hard platen, soft platen, hard caliper through a 12-sheet stack, and calculation of the Setterholm MD effective thickness $t_e$ calculated using $E_{MD}$ is shown to have a strong correlation with the soft caliper $t_s$ with $t_e = 0.9946 t_s + 0.0054$ mm. Error bars represent 95% confidence intervals about mean values.

Elastic modulus
The elastic modulus calculated from Eq. (8) for seven different grades of paper is shown in Fig. 3. The noncellulosics form a much less oriented sheet than the cellulosic papers. Figure 3B also shows the relationship between the basis weight and the squareness (MD/CD modulus ratio) of the sheet. The newsprint sample (D) and lightweight sample (E) are highly oriented because manufacturing on high-speed machines generally produces a stronger fiber orientation. Lin-
Bending stiffness

Figure 4 shows the bending stiffness of seven different samples measured by three different methods: L&W with 5° degree bend angle, L&W with 30° degree bend angle, and Taber stiffness with 15° degree bend angle, plotted against the theoretical bending stiffness calculated using the measured elastic modulus and soft-platen caliper. The calculated bending stiffness is assumed to be free of measurement artefacts compared with the direct measurement of bending stiffness. All three methods correlate well with theory. The agreement is quantified by regression constants for the three methods, which are summarized in Table III. Deviation of the experimental values from theoretical values is illustrated in Fig. 5. The bending stiffnesses for lightweight grades are systematically underestimated by 30% by all three methods because, in the lower ranges, the required short spans and high bending strains necessary to record a measurement lead to plastic behavior of the test specimen.

Figure 6 shows ultrasonic measured stiffnesses $E_{ij}t_{eq}$ for the seven samples, and compares it with tensile stiffnesses measured on the Instron testing frame. The tensile stiffness values from the two sets of measurements correlate well, with a ratio of ~1.47 between them. The observed difference can be attributed to the viscoelastic nature of paper, meaning that decreased relaxation time should yield larger stiffnesses than mechanical counterparts [12]. Comparison of individual moduli showed a wider scatter than the comparison of stiffnesses (see Fig. 7). The LWC sample’s ultrasonic elastic modulus was farthest away from the fitted line, which is probably attributable to its high content of clay filler and coating, clay having a reported shear modulus of 22 GPa [14].
DISCUSSION

**Basis weight**

Highly accurate measurements on paper are often difficult because of the high degree of inhomogeneity of the sample. The measurements presented here for basis weight were precise only to two significant figures because of the limitations of measuring sample area. One suggested solution may be to scan each basis-weight test specimen into a computer and then to use image analysis to measure the digitized area. A common table-top scanner can provide 50-micron pixel resolution, so that over a typical specimen area of 400 cm$^2$ or more, the digitized area could be measured to five significant figures.

Intra-sample variations in basis weight in the paper sample set were in the order of 1%–2%, so the expense of higher-precision area measurement will not significantly reduce this variability. For the most part, the measurements of the coefficient of variation reported here (Fig. 1) decreased with increasing sample basis weight, which is consistent with Fellers’ observations [6].

**Caliper**

The estimation of caliper from a stack of sheets usually gave a larger caliper value than either the soft-platen or the hard-platen caliper measurements, with the exception of the B sample. This indicates that variations in formation of the individual sheets are additive and make the sheet effectively thicker in a stack. It is also possible that, over time, the viscoelastic nature of paper will enable the sheets to deform to accommodate variations in formation, reducing the effective caliper closer to that of the individual sheets.

One problem in the analysis was that the errors between hard-platen and soft-platen caliper measurements were inconsistent. For smooth-surface materials, the hard caliper method would be preferred over the soft caliper method because of its greater accuracy, because the deformation of the neoprene platen during measurement would introduce a variability not present in the sheet. However, a rough surface is more amenable to the soft-platen method because in this case topographical surface undulations have less additive effect. The prominent surface features indent into the neoprene platen upon application of the platen measurement pressure, which results in a true representation of the mean caliper.

**Elastic modulus**

**Figure 8** shows the elastic modulus of the cellulosic samples as a function of basis weight and density. Generally, unless there are unusual process changes, strength properties are
expected to increase with basis weight. The MD elastic modulus appears to increase with density, while the CD elastic modulus has a stronger correlation with basis weight.

**Bending stiffness**

The two-point L&W tester was used to draw comparisons between bending stiffnesses for 5° and 30° deflections. For most of the cellulosics, the stiffnesses at 30° deflection were lower than those at 5°. The only exception was the CD stiffness of the newsprint sample. This sample also had the lowest stiffness measured and may be beyond the limits of the L&W tester’s sensitivity. Among the noncellulosics, we observed the opposite trend. The 30° deflections indicated higher bending stiffnesses than those at 5°. The lower value for paper may be due to a failure mode not present in nonfibrous noncellulosic sheets.

Among the cellulosic sheets, the Taber test tended to give readings within one standard deviation of the 30° L&W reading. The most marked departure from this trend was the linerboard sample, where the Taber test values fell right between the 5° and 30° L&W measurements, as might be expected. From this observation, it can be assumed that for high basis weights (> 200 g/m²), the L&W bending-stiffness tester will give results comparable with those from the Taber tester for similar deflections. For lower basis weights (< 200 g/m²), greater deflections on the L&W instrument will give results comparable to those from the Taber tester. However, smaller deflections on the L&W will measure larger stiffness values than the Taber for stiffnesses greater than 0.1 Taber units (0.02 mN-m). In addition to a range of available deflections, the L&W also has the advantage of a range of adjustable test spans. For low-basis-weight papers, this enables a span to be selected to obtain the smallest possible deflection and loss to plasticization [6], thus leading to an optimal agreement with calculated bending stiffness.

**Ultrasonic measurements**

We compared the current measurements to those reported by Baum et al. [15] as shown in Fig. 9 and Table IV. Only four of the seven samples studied here had basis weights comparable to the samples tested in that report. Even so, the present results are similar to those reported in [15], albeit with wide scatter. This can be attributed to the fact that different samples were tested in this paper.

<table>
<thead>
<tr>
<th>Comparison Sample</th>
<th>Basis Wt. (g/m²)</th>
<th>Density (kg/m³)</th>
<th>E₁₁ (MPa)</th>
<th>E₂₂ (MPa)</th>
<th>G₁₂ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Linerboard</td>
<td>212</td>
<td>766</td>
<td>782</td>
<td>7233</td>
</tr>
<tr>
<td>B</td>
<td>Medium</td>
<td>130</td>
<td>679</td>
<td>538</td>
<td>4683</td>
</tr>
<tr>
<td>E</td>
<td>LWC</td>
<td>47</td>
<td>1057</td>
<td>762</td>
<td>8747</td>
</tr>
<tr>
<td>G</td>
<td>Handsheet</td>
<td>156</td>
<td>1610</td>
<td>575</td>
<td>4677</td>
</tr>
</tbody>
</table>

**IV. Comparison of ultrasonic measured moduli with similar values reported by Baum [15] (highlighted). Sample E is LWC, which is paired with a low-basis-weight linerboard. Sample G, a synthetic paper, is paired with a handsheet of roughly equivalent basis weight.**
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Ultrasonic measurements ensure the validity of mechanical measurements in terms of the relative ranking of the moduli or tensile stiffnesses of a sample set.

CONCLUSIONS
The bending stiffnesses of papers ranging from 48 g/m² to 212 g/m² basis weight have been tested by four different means. Good agreement was generally found between the Taber and L&W methods. The decrease in bending stiffness for cellulosics at higher bending angles would suggest that there is some plasticization of the beam-like test specimen, because this phenomenon is not observed in the noncellulosic samples. In addition, the L&W bending-stiffness instrument has the advantage of an adjustable span, which enables the user to make the best choice of span and deflection angle for a given basis weight. The selection of bending-stiffness test parameters can be checked by comparison of measured values with those calculated using measurements of the tensile stiffness and soft-platen caliper of the test sample. Too short a span or too great a deflection can result in testing the specimen in the nonlinear region of its stress-strain behavior, leading to plastic deformation and an underestimate of the bending stiffness, especially for low-basis-weight papers.

The coefficient of variation of basis weight was reported, showing that for low basis weights, the coefficient of variation was unchanged over the seven test specimens. A method was discussed whereby a more precise basis weight could be measured more easily using a flatbed scanner. In addition, a new method of caliper measurement was suggested, involving measurement of multiple sheets to obtain an average over the stack.

In-plane ultrasonic measurements can be used to verify mechanical measurements of tensile stiffness and modulus for a sample set from a relative ranking perspective. Because paper is viscoelastic, the high-frequency measurement produces absolute values 40% to 50% greater than those measured by standard mechanical methods. The advantage is that this method is nondestructive and comparatively faster than mechanical methods requiring sample cutting and handling. Moreover, ultrasonic measurements of the specific moduli $E_{MD,CD}/\rho$ are now readily commercially available and can be implemented in automated inline testers. TJ

LITERATURE CITED

INSIGHTS FROM THE AUTHORS
A true value of the bending stiffness of paperboard is required to calculate the buckling load of panels made from that material. Comparison of Taber measurements of bending stiffness with those obtained from other instruments has revealed discrepancies. This prompted us to calculate bending stiffness from simple mechanics for a variety of papers, which made apparent the interrelationships among various measurements and how they can be used to ratify each other. Previous results from the Forest Products Laboratory (FPL—Setterholm) and the Institute for Paper Chemistry (IPC—Baum) regarding the significance of soft-platen caliper were used to validate measurements from two popular instruments for bending stiffness. We hope the results of this research can serve as a guide for qualifying and interpreting bending stiffness measurements.

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