The BCT of Copy Paper Boxes – Applying McKee’s Formula

Presented by:

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Problem definition

• Copy paper supplier/end-user (IPST testing services client) wishes to know who provides boxes of what strength for xerography copy papers – world-wide

• Supplier/end-user has limited/minimal testing capability but needs some (easy) means to predict BCT for stacking strength

• Office practice is to store copy paper pallets (5 x 5 x 5 boxes high) stack failure occasionally occurs, requiring restacking
Does BCT of copy paper boxes matter?

Typical storage conditions at IPST

Pallets at Office Depot are 5 high

Close up of bottom edge shows edge-roll damage
What’s different about Copy Paper Boxes

• Stacking strength usually not a concern for design since the paper reams provide vertical support

• Design is “tray with cover” style, the board is used to “wrap” the product, side panels are often oriented with vertical load in the MD

• Footprint (box length and width) is constant (for this study), panel structure and flap designs vary
Typical 5 pack copy paper box with the load along CD of the panels

A copy paper box loaded with 5 reams of paper weights ~ 26 lbs. BCT of an empty box can be ~ 120 lbs...So in a stack of 5 boxes high...
Boxes with loads along the MD
Copy paper boxes reassembled and ready for BCT

23 of the 42 boxes are constructed so the side panels have stack load in the board CD
42 different copy paper boxes – can the McKee formula be used to predict BCT?

- Z (perimeter) is constant (41 inches)
- Load supporting panels are oriented MD or CD
- Boxes have a tray lid
- Boxes for lab study were all supplied post-use
Approach

- Boxes collected by client were reassembled with hot melt at IPST
- Boxes were tested for BCT with their trays on
- Box bottom flaps were tested for caliper and ECT (T 839) in MD or CD
- Simplified McKee equation applied to fit the data
BCT of assembled boxes

Boxes were tested empty with the top lid trays on
ECT in the board MD is a lot less

Medium fluting contributes to ECT when loaded in CD

When load is along the MD, the board fails by buckling

• B flute copy paper box board ECT along CD is 36lb/in, along the MD it is 16 lb/in

• One box with 5 reams of paper weighs 26 lbs, many bottom boxes in a pallet stack are at their BCT failure load!!
Simplified form of the McKee model

\[ BCT = 5.87 \times ECT \times \sqrt{t \times Z} \]

- Equation derivation assumes:
  - Square footprint box
  - No shear
  - Boxes are high enough for panel buckling
  - Panel buckling is proportional to “sandwich beam” bending stiffness which is proportional to tensile stiffness of the liners
  - Tensile stiffness of the liners is proportional to ECT

- But, ease of use makes this a preferred model, accuracy sufficient for many estimates
Fitting a “McKee” BCT to the data

\[ BCT_{(lbs)} = C \times ECT_{(lb/in)}^a \times t_{(in)}^b \times \sqrt{41} \]

• Assume basic form of the simplified McKee equation can describe the BCT of copy paper boxes

• Fit data using \( ECT \) and \( t \) measured from the bottom panels

• Excel® Solver function is used to minimize the difference between the fitted McKee and actual BCT by iteratively changing \( C, a, \) and \( b \)
Fitted form of McKee for Copy paper boxes

\[ BCT = 6.03 \times ECT^{0.802} t^{0.844} \]

Graph showing the relationship between Actual BCT (lbs) and Predicted from fitted McKee (ECT, t) or Regular McKee equation. The graph includes fitted points and a trend line with an R^2 value of 0.6957.

Legend:
- Fitted Model
- Regular McKee model
- Linear (Fitted Model)
Actual BCT vs Model Predicted values

• Original McKee equation generally predicts values that are too high, average error = 52%

• Fitted modified McKee model predicts values closer to actual, average error = 25%

• Client can now predict the BCT from measurements of ECT (in CD or MD) and caliper of the board
Can the TSO replace ECT testing??

1. Elastic modulus $E$, density $\rho$ and speed of sound $V$:

$$E \cong \rho V^2$$

$V^2$ is called $TSI_{CD}$ or $TSI_{MD}$

$$E \times t = \rho t \times V^2 = \text{basis weight} \times V^2$$

2. So, if we measure the basis weight $BW$ and the speed of sound squared $V^2$ - we get the tensile stiffness $Et$

3. McKee (1963) assumed and showed the proportionality of tensile stiffness $Et$ to $ECT$ of the board - so why not measure $V^2$ of corrugated board instead of ECT? *No cutting, no waxing, no clamping...*
Using the TSO to predict ECT of corrugated boards – an idea:

\[
ECT \approx 0.7 \left( 2 \times SCT_{\text{liner}} + 1.42 \times SCT_{\text{medium}} \right)
\]

1. \(\text{TSI}_{\text{CD}} = E_{\text{CD}} \times t = \text{basis weight} \times V_{CD}^2\)

2. Wavelength \(\lambda\) of ultrasonic sound waves of 100 kHz exceeds thickness \(t\) of boards:

\[
\lambda = V_{CD}/100 \text{ kHz} = 2.7 \text{ km/s}/100 \text{ KHz} \approx 2.7 \text{ cm}
\]

3. Sound waves (longitudinal) propagate along the board through the whole board

4. Therefore \(ECT \approx (\text{board basis weight} \times \text{TSI}_{\text{CD}})\)
ECT of lab-made A flute boards with different weights of medium, all have the same liner weight

\[ ECT (kN/m) = 1.34 \ (E_{CD}t) + 2.94 \pm 0.36 \]

\[ R^2 = 0.84 \]

– here the TSO detects changes in medium strength
Using the TSO tester for board

- The board is laid flat, the MD of the measurement is along the width of the TSO
- Single measurements are made (~3 seconds!), get polar plot and TSI_MD and TSI_CD

Use TSI_MD or TSI_CD (units are equivalent to km/s²) pending on box load orientation times board basis weight
Now back to the Copy Paper Box set from Southeast Asia:
Can Ultrasonic Testing Replace ECT? (maybe !)

1. Shipped boxes submitted by client were reassembled, tested for BCT, bottom flaps for ECT

2. Side panels were ultrasonically tested after (!) BCT

3. Used $V_{CD}^2 \times BW$ for those boards where vertical loading of the side panels is in the CD

4. $V_{CD}^2 \times BW$ correlated well with ECT
Correlation of Copy Paper box ECT with ultrasonics

ECT = 24.8(TSI_CD x BW) - 21.0
$R^2 = 0.76$

Average variation in ECT $\sim 8\%$, variation in TSI x BW $\sim 2\%$
Summary

1. A modified simplified McKee equation can be used to predict the BCT of copy paper boxes using board caliper and board ECT.

2. Copy paper boxes with vertical load in the CD of the board have higher BCT, (of course!)

1. Ultrasonic $TSI_{CD}$ with board basis weight predicts $ECT$ of boards loaded in the CD with less variation – can be a new convenient quality control check i.e., no cutting, no sample prep...let’s try it out there!!

Contemplating a world without damaged boxes ?..
Thank you! Send questions, comments, testing samples to: Roman@gatech.edu

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Yes !! Still at it !!

“serving the paper industry since 1929…to survive is to do research, but to thrive is to implement…”
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Thank you

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